An-Najah National University

Faculty of Engineering

Industrial Engineering Department

Course :

Quantitative Methods (65211)

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Chapter 2

Probability

2-1 SAMPLE SPACES AND EVENTS

2-1.1 Random Experiments

❖ In Conducting Experiments, the results can differ slightly because of small variations in variables that are not controlled in our experiment.

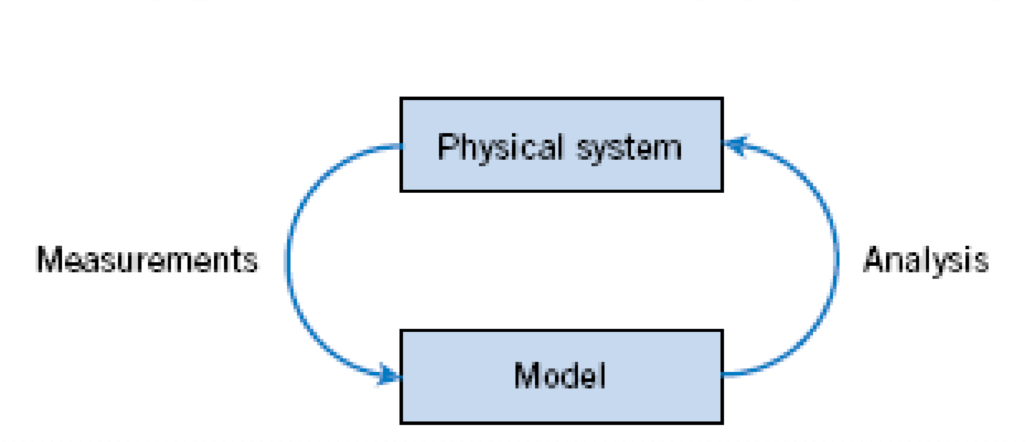
➢ Example:

Thin Copper Wire Current Measurement

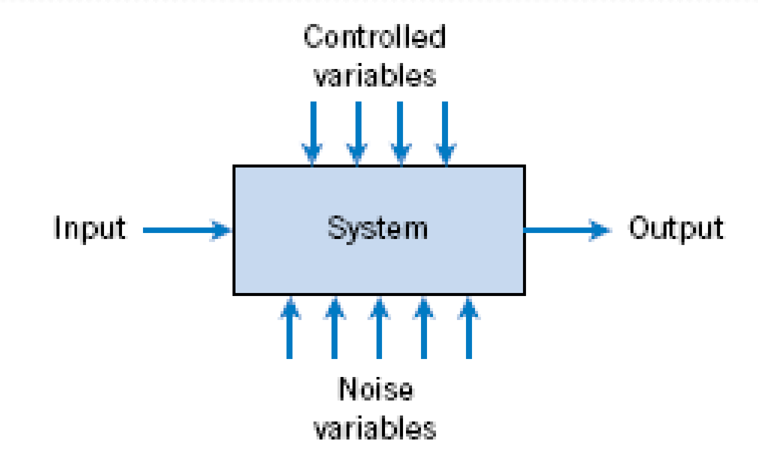
Sources of Variability: (changes in ambient temperatures, slight variations in gauge and small impurities in the chemical composition of the wire if different locations are selected, and current source drifts).

So that, Experiment $\longrightarrow$ **Random Component**

❖ we can use the model to understand, describe, and quantify important aspects of the physical system and predict the response of the system to inputs.



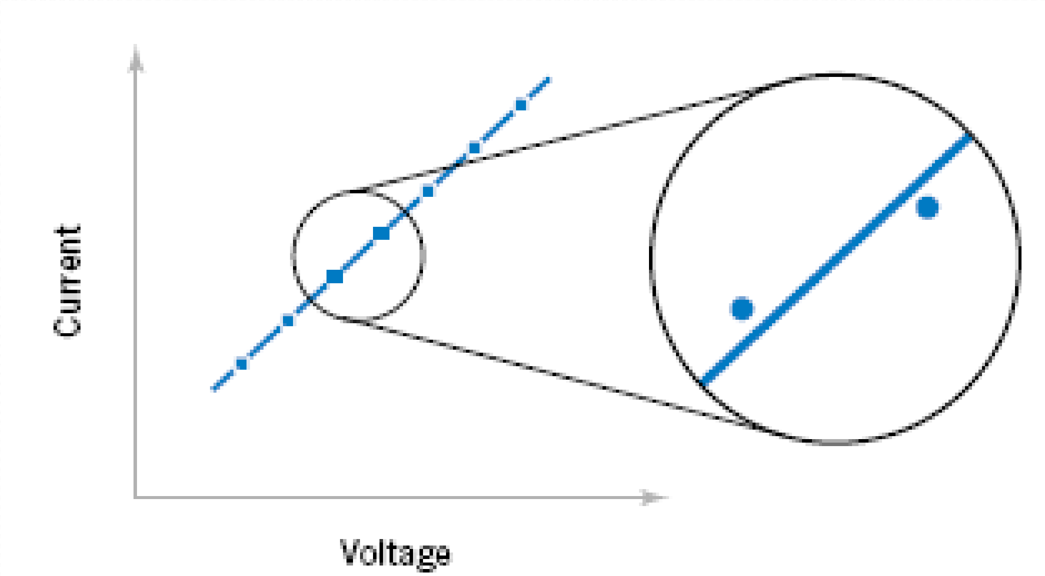
❖ The following figure displays a model that incorporates uncontrollable inputs (noise) that combine with the controllable inputs to produce the output of our system.



❖ **Random Experiment** Definition:

An experiment that can result in different outcomes, even though it is repeated in the same manner every time, is called a random experiment.

➢ **Example:** Again, Measuring current flow in a thin copper wire.



A closer examination of the system identifies deviations from the model.

2-1.2 Sample Spaces

❖ **Sample Space** Definition:

The set of all possible outcomes of a random experiment is called the **sample space** of the experiment. The sample space is denoted as *S*.

❖ It is often defined based on the objectives of the analysis.

➢ **Example:**

Molded Plastic Part, such as a connector.

Space 1: Thickness (Always Positive)

$$S = R^+ = \{x \mid x > 0\}$$

Space 2: Thickness (Between 10 and 11 mm)

$$S = \{x \mid 10 < x < 11\}$$

Space 3: low, medium, or high for thickness

$$S = \{low, medium, high\}$$

Space 4: Conformance to manufacturing specifications

$$S = \{yes, no\}$$

❖ Types of Sample Spaces:

1- Discrete: consists of a finite or countable infinite set of outcomes.
2- continuous: contains an interval (either finite of infinite) of real numbers.

➢ **Example:**

For two Molded Plastic Parts or connectors.

$$S = R^+ \times R^+$$

$$S = \{yy, yn, ny, nn\}$$

If we are only interested in the number of conforming parts in the sample

$$S = \{0, 1, 2\}$$

❖ As another example, consider an experiment in which the thickness is measured until a connector fails to meet the specifications.

$$S = \{n, yn, yyn, yyyn, yyyyn, \text{and so forth}\}$$

➢ **Example:**

Batch consists of three items {a, b, c}

Without Replacement:

$$S_{\text{without}} = \{ab, ac, ba, bc, ca, cb\}$$

With Replacement:

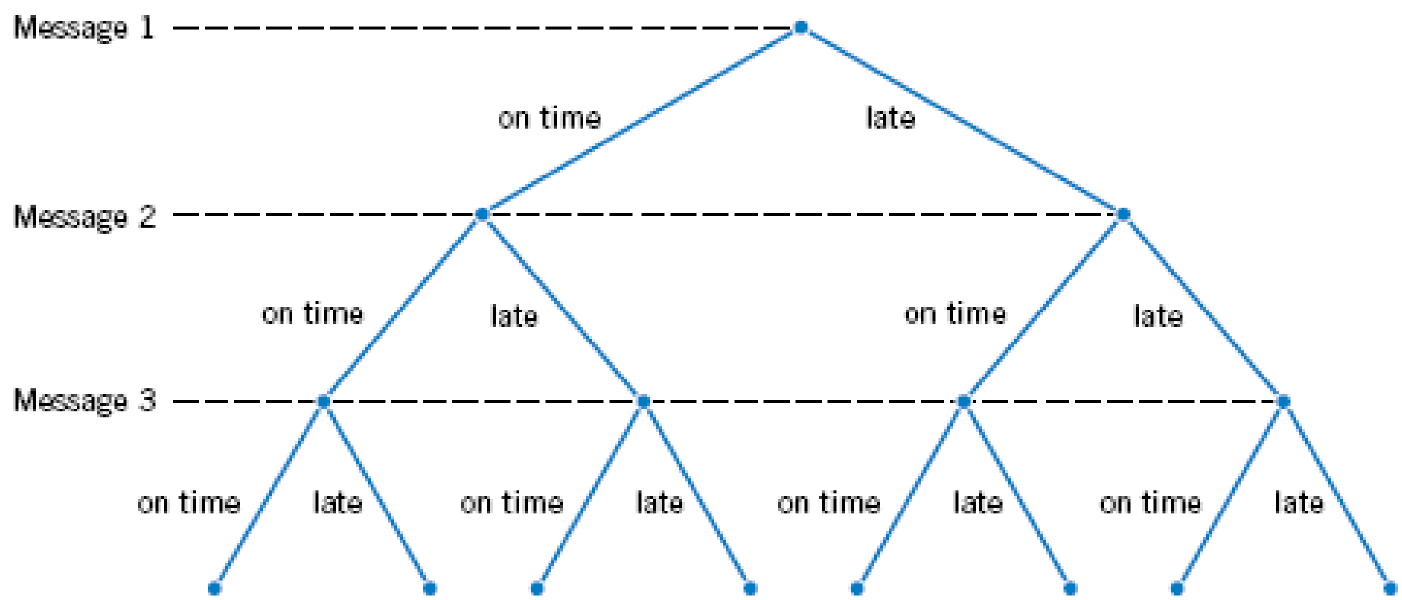
$$S_{\text{with}} = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$$

The unordered description of the sample space is $\{\{a, a\}, \{a, b\}, \{a, c\}, \{b, b\}, \{b, c\}, \{c, c\}\}$.

❖ Tree Diagrams: Can be used to describe sample spaces graphically.

➢ **Example:**

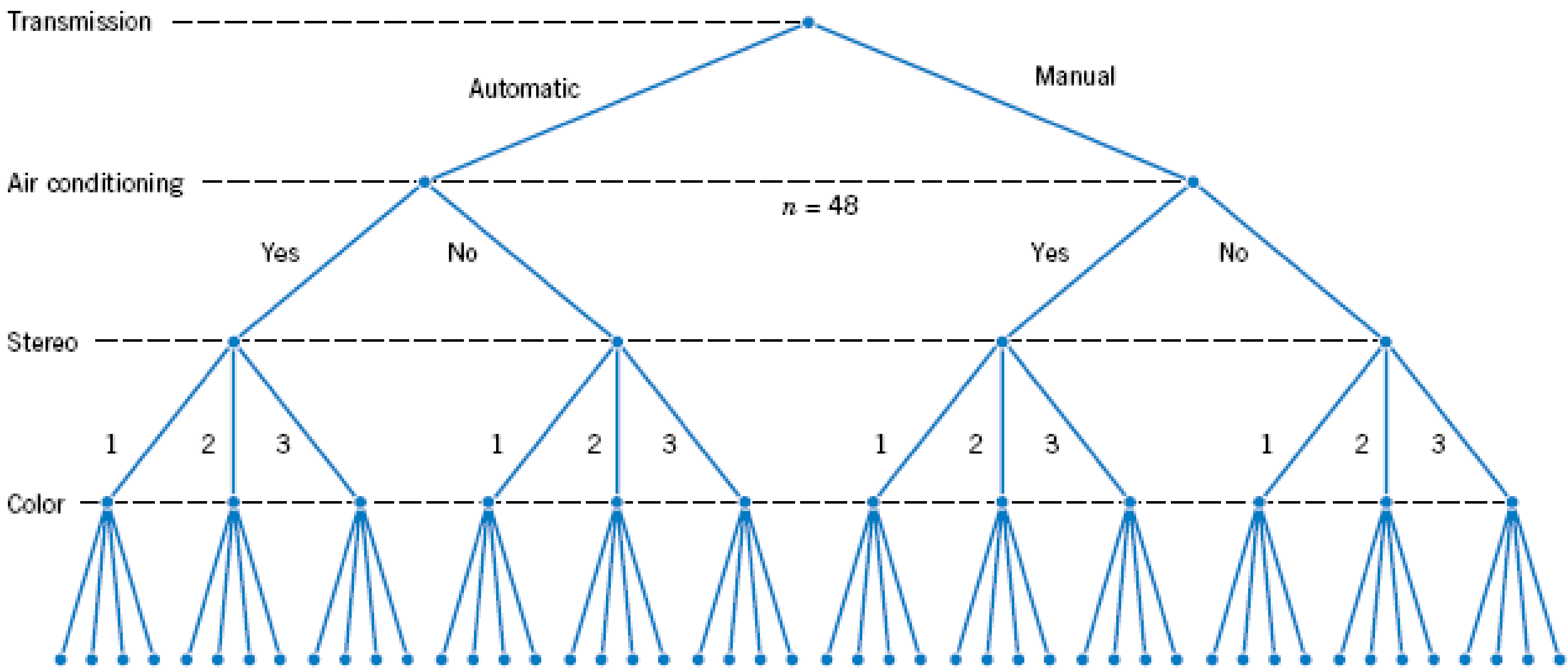
Each message in a digital communication system is classified as to whether it is received within the time specified by the system design.



➢ Example:

An automobile manufacturer provides vehicles equipped with selected options. Each vehicle is ordered

- With or without an automatic transmission
- With or without air-conditioning
- With one of three choices of a stereo System
- With one of four exterior colors



Transmission
Automatic
Manual
Air conditioning
n = 48
Yes
No
Yes
No
Stereo
1
2
3
1
2
3
1
2
3
1
2
3
Color

2-1.3 Events:

❖ **Event** Definition:

An **event is a subset of the sample space of a random experiment.**

❖ **We** can also be interested in describing new events from combinations of existing events.

1- The **union of two events is the event that consists of all outcomes that are contained** in either of the two events. We denote the union as $E_1 \cup E_2$

2- The **intersection of two events is the event that consists of all outcomes that are** contained in both of the two events. We denote the intersection as $E_1 \cap E_2$

3- The **complement of an event in a sample space is the set of outcomes in the sample** space that are not in the event. We denote the component of the event *E as* E'

➢ Example:

Consider the sample space S = {yy, yn, ny, nn}

Then:

-At lest one part conforms	E1 = {yy,yn,ny}
- Both Parts do not conform	E2 = {nn}
-Null set	E3 = ϕ
- if E5 = {yn, ny, nn}	

So that,

E1 U E5 = S   E1 ∩ E5 = {yn, ny}   E1' = {nn}

➢ Example:

Measurements of the Thickness of a plastic connector might be modeled with the sample space S = R^+

$$E_1 = \{x \mid 1 \le x < 10\} \quad \text{and} \quad E_2 = \{x \mid 3 < x < 118\}$$

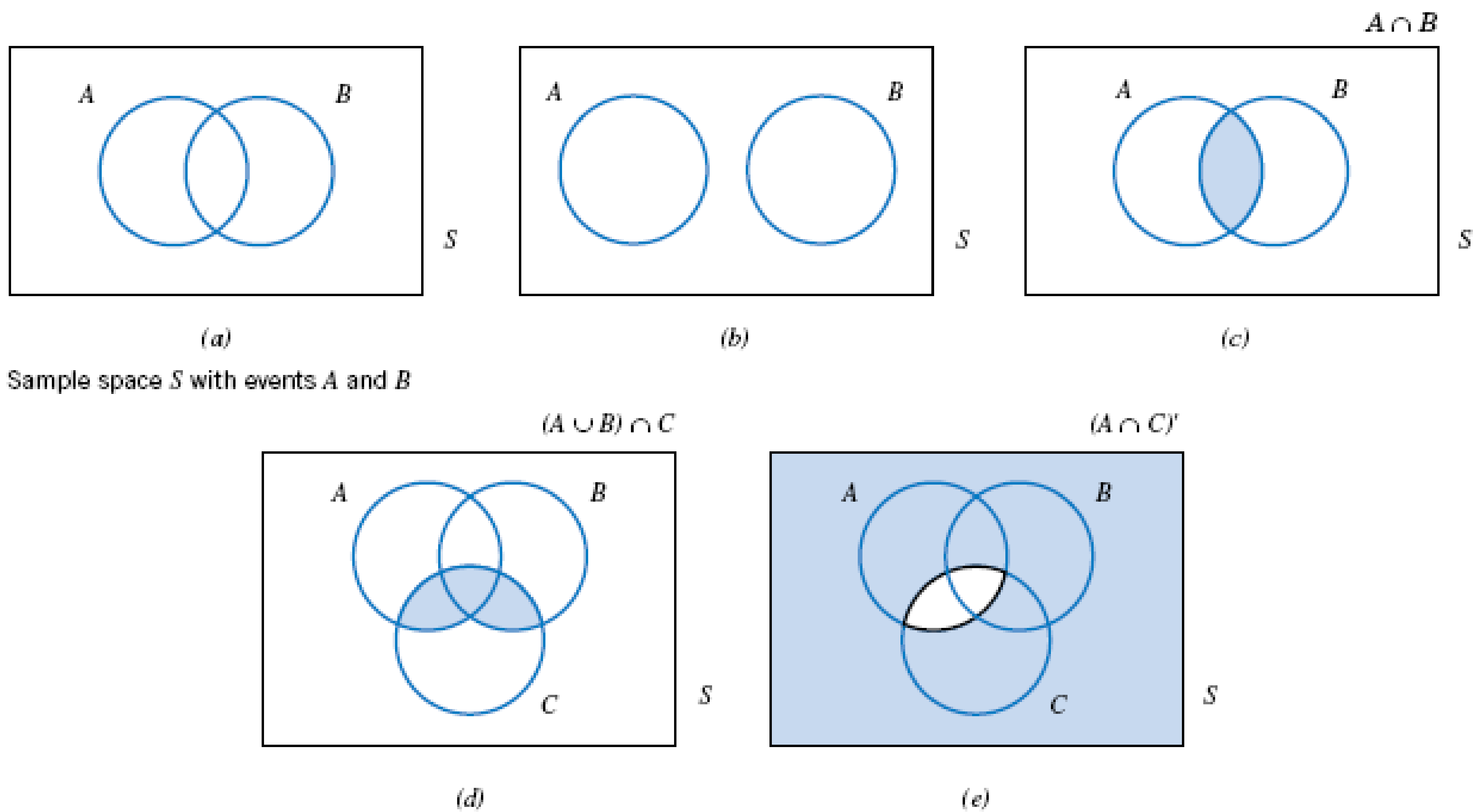
Then,

$$E_1 \cup E_2 = \{x \mid 1 \le x < 118\} \quad \text{and} \quad E_1 \cap E_2 = \{x \mid 3 < x < 10\}$$

Also,

$$E_1' = \{x \mid x \ge 10\} \quad \text{and} \quad E_1' \cap E_2 = \{x \mid 10 \le x < 118\}$$

Venn Diagrams:



❖ **Mutually Exclusive** Events:

Two events, denoted as E1 and E2, such that

$$E1 \cap E2$$

are said to be **mutually exclusive**.

❖ Additional results involving events are summarized below. The definition of the complement of an event implies that

$$(E')' = E$$

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad \text{and} \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

DeMorgan's laws imply that

$$(A \cup B)' = A' \cap B' \quad \text{and} \quad (A \cap B)' = A' \cup B'$$

Also, remember that

$$A \cap B = B \cap A \quad \text{and} \quad A \cup B = B \cup A$$

2-1.4 Counting Techniques:

❖ Methods for counting the number of outcomes in the sample space and various events to be used in analyzing random experiments.

❖ **Multiplication Rule** (for Counting Techniques):

If an operation can be described as a sequence of *k* steps, and

if the number of ways of completing step 1 is $\mathbf{n_1}$, and
if the number of ways of completing step 2 is $\mathbf{n_2}$ for each way completing step 1, and
if the number of ways of completing step 3 is $\mathbf{n_3}$ for each way completing step 2, and so forth,

The total number of ways of completing the operation is

$$\mathbf{n_1} \text{ X } \mathbf{n_2} \text{ X } \ldots \text{ X } \mathbf{n_k}$$

- **Example:**

Gear housing casting design 4 types of fasteners, 3 different bolt lengths, and 3 different bolt locations.

# of designs = 4 X 3 X 3 = 36

Permutations:

The number of permutations of n different elements is ***n*!** where

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

It calculates the number of ordered sequences of elements of a set.

- **Example:**

S = {a, b, c}.

3! = 3 X 2 X 1 = 6.

❖ Permutations of subsets:

The number of permutations of subsets of ***r*** elements selected from a set of ***n*** different elements is

$$P_r^n = n \times (n - 1) \times (n - 2) \times \ldots \times (n - r + 1) = \frac{n!}{(n - r)!}$$

➢ Example:

A printed circuit board has 8 different locations in which a component can be placed, if four different components are to be placed on the board, how many different designs we have?

$$P_4^8 = 8 \times 7 \times 6 \times 5 = \frac{8!}{4!} = 1680$$

❖ Permutations of Similar Objects:

The number of permutations of $n = n_1 + n_2 + n_3 + \ldots + n_r$ objects of which n_1 are of one type, n_2 are of a second type, …, and n_r are of an rth type is

$$n! \,/\, (n_1!\; n_2!\; n_3!\; \ldots\; n_r!)$$

➢ **Example:**

Consider a machining operation in which a piece of sheet metal needs two identical diameter holes drilled and two identical size notches cut.

# of possible sequences = 4! / (2! 2!) = 6

❖ Combinations:

The number of subsets of size r that can be selected from a set of n elements is

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

The Order is not Important.

➢ Example:

A printed circuit board has eight different locations in which a component can be placed. If five identical components are to be placed on the board, how many different designs are possible?

# of possible sequences = 8! / (5! 3!) = 56

2-2 INTERPRETATION OF PROBABILITY

2-2.1 Introduction.

❖ **Probability** is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur.

❖ Higher numbers indicate that the outcome is more likely than lower numbers.

❖ Range: ***0*** → ***100%***

❖ **More Interpretations**:

✓ Degree of belief, that the outcome will occur
✓ Based on the conceptual model of repeated replications of the random experiment.

$$\sum \text{Probabilities of all outcomes} = 1$$

❖ <u>Equally Likely Outcomes:</u>

Whenever a sample space consists of ***N*** possible outcomes that are <u>**equally likely**</u>, the probability of each outcome is ***1/N***.

➢ Example:

Batch of diodes contains 100 diode.

So that, the probability of choosing each one = 1/100 = 0.01 (constant)

❖ <u>Probability of an Event:</u>

For a discrete sample space, the *probability of an event* ***E***, denoted as ***P(E)***, equals the sum of the probabilities of the outcomes in ***E***.

➢ **Example:**

Batch of 100 small balls, 30 are red and 70 are green.

Let E denote a subset of 30 red balls.

So that P (E) = 0.01X30 = 30/100 = 0.3.

➢ **Example:**

S = {a, b, c, d}, such as P(a) = 0.1, P(b) = 0.3, P(c) = 0.5, and P(d) = 0.1.
A = {a, b}, B = {b, c, d}, and C = {d}. Then

P(A) = 0.1+0.3 = 0.4

P(B) = 0.3+0.5+0.1 = 0.9

P(C) = 0.1

P(A') = 1-0.4 = 0.6

P(B') = 1-0.9 = 0.1

P(C') = 1-0.1 = 0.9

P(A∩B) = P(b) = 0.3

P(AUB) = P(S) = 1

P(A∩B) = P(ϕ) = 0

2-2.2 Axioms of Probability.

❖ The probabilities in any random experiment must satisfy these axioms.

❖ The axioms ***do not*** determine probabilities; the probabilities are assigned based on our knowledge of the system under study.

❖ **Axioms of Probability:**

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

1- $P(S) = 1$
2- $0 \leq P(E) \leq 1$
3- For two events E_1 and E_2 with $E_1 \cap E_2 = \phi$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

❖ If E_1 is contained in the event E_2, then

$$P(E_1) \leq P(E_2)$$

➢ **Question 2-57.**

In a NiCd battery, a fully charged cell is composed of Nickelic Hydroxide. Nickel is an element that has multiple oxidation sates that is usually found in the following states:

Nickel Charge	Proportions found
0	0.17
+2	0.35
+3	0.33
+4	0.15

a) Prob. that a cell has at least one of the (+) nickel charged options?

Prob. = P(+2) + P(+3) + P(+4) = 0.35 + 0.33 + 0.15 = 0.83

b) Prob. that a cell is not composed of a positive nickel charge greater that +3?

Prob. = 1- P(+3) = P(0) + P(+2) + P(+3) = 1 – 0.15 = 0.17 + 0.35 + 0.33 = 0.85

2-3 ADDITION RULES:

❖ It considers joint events (Unions, Intersections, and Complements). There probabilities can be determined from the probabilities of the individual events that comprise them.

❖ **Probability of a Union:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

❖ **Mutually Exclusive Events:**

If A and B are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B)$$

- **Example:**

Wafers in Semiconductors Manufacturing Classified by Contamination and Location.

Location in Sputtering Tool			
Contamination	Center	Edge	Total
Low	514	68	582
High	112	246	358
Total	626	314	

Let H: event that wafer contains high levels of contamination.
Then $P(H) = 358/940$

Let C: event that wafer is in the center of spluttering tool.
Then $P(C) = 626/940$

$P(H \cap C) = 112/940$ But $P(H \cup C) = (514+112+246)/940 = \underline{\boldsymbol{872/940}}$

$P(H \cup C) = P(H) + P(C) - P(H \cap C) = (358/940) + (626/940) - (112/940) = \underline{\boldsymbol{872/940}}$

➢ Example:

Another Classification for Wafers in Semiconductors Manufacturing Classified by Contamination and Location.

Number of Contamination Particles	Center	Edge	Totals
0	0.30	0.10	0.40
1	0.15	0.05	0.20
2	0.10	0.05	0.15
3	0.06	0.04	0.10
4	0.04	0.01	0.05
5 or more	0.07	0.03	0.10
Totals	0.72	0.28	1.00

1) What is the probability that a wafer was either at the edge or that it contains four or more particles?

E_1: wafer contains four or more particles.
E_2: wafer is at the edge.

$P(E_1 \cup E_2) = (0.05+0.1) + (0.28) – (0.01+0.03) = 0.39$

Number of Contamination Particles	Center	Edge	Totals
0	0.30	0.10	0.40
1	0.15	0.05	0.20
2	0.10	0.05	0.15
3	0.06	0.04	0.10
4	0.04	0.01	0.05
5 or more	0.07	0.03	0.10
Totals	0.72	0.28	1.00

2) What is the probability that a wafer contains less than two particles or that it is both at the edge and contains more than four particles?

E_1: wafer contains less than two particles.
E_2: wafer is both from the edge and contains more that four particles.

$P(E_1 \text{ U } E_2) = ?????$
$P(E_1) = (0.4+0.2) = 0.6$ $P(E_2) = 0.03$
E_1 and E_2 are mutually exclusive because $P(E_1 \cap E_2) = 0$

So that, $P(E_1 \text{ U } E_2) = 0.6 + 0.03 - 0 = 0.63$

❖ Three or More Events:

For two Events:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

❖ Mutually Exclusive Events:

A collection of events, $E_1, E_2, \ldots, E_k$, is said to be **mutually exclusive** if for all pairs,

$$E_i \cap E_j = \varnothing$$

For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \ldots \cup E_k) = P(E_1) + P(E_2) + \ldots P(E_k)$$

➢ Example:

X denotes the pH of a sample.

Consider the event that X is greater than 6.5 but less than or equal to 7.8.

This probability is the sum of any collection of mutually exclusive events with union equal to the same range for X.

$$P(6.5 < X \leq 7.8) = P(6.5 < X \leq 7.0) + P(7.0 < X \leq 7.5) + P(7.5 < X \leq 7.8)$$

Another example is

$$\begin{aligned} P(6.5 < X \leq 7.8) = P(6.5 < X \leq 6.6) &+ P(6.6 < X \leq 7.1) \\ &+ P(7.1 < X \leq 7.4) + P(7.4 < X \leq 7.8) \end{aligned}$$

➢ Question 2-71.

The Analysis of shafts for a compressor is summarized by conformance to specifications.

		roundness conforms	
		yes	no
surface finish	yes	345	5
conforms	no	12	8

(a) P (the shaft conforms to surface finish requirements) = (345+5)/370 = 350/370

(b) P (conforms to surface finish or to roundness) = (350/370) + (357/370) – (345/370) = 362/370

(c) P (conforms to surface finish or does not conform to roundness) = (350/370) + (13/370) – (5/370) = 358/370

(d) P (conforms to both) = 345/370

2-4 CONDITIONAL PROBABILITY:

❖ Sometimes probabilities need to be reevaluated as additional information becomes available.

❖ The Probability of an event ***B*** under the knowledge that the outcome will be in event ***A*** is denoted as

P(B|A)

And this is called **Conditional Probability** of ***B*** **GIVEN** ***A.***

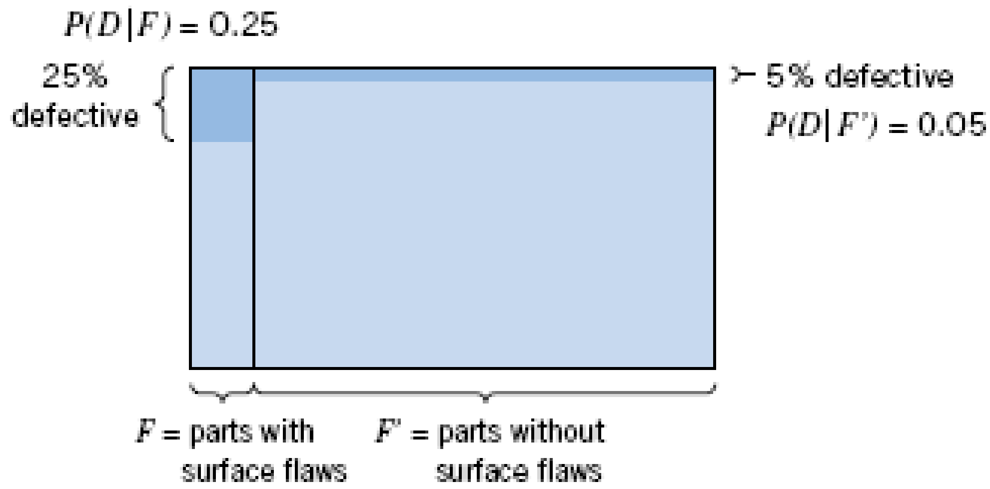
➢ **Example:**

In a manufacturing process, 10% of parts contain visible surface flaw.
25% of the parts with surface flaws are defective parts.
5% of parts without surface flaws are defective parts.

Probability of defective part depends on the absence of presence of a surface flaw.

D: The part is defective.
F: The part has a surface flaw.

$P(D|F) = 0.25$ $P(D|F') = 0.05$



400 parts classified by surface flaws and as (functionally) defective.

		Surface Flaws		
		Yes (event F)	No	Total
Defective	Yes (event D)	10	18	28
	No	30	342	372
	Total	40	360	400

$P(D|F) = 10/40 = 0.25$

$P(D|F') = 18/360 = 0.05$

❖ Conditional Probability:

The **conditional probability** of an event B given an event A, denoted as $P(B|A)$, is

$$P(B|A) = P(A \cap B)/P(A) \qquad (2\text{-}5)$$

for $P(A) > 0$.

This definition can be understood in a special case in which all outcomes of a random experiment are equally likely. If there are ***n*** total outcomes,

$$P(A) = (\text{number of outcomes in } A)/n$$

Also,

$$P(A \cap B) = (\text{number of outcomes in } A \cap B)/n$$

Consequently,

$$P(A \cap B)/P(A) = (\text{number of outcomes in } A \cap B)/\text{number of outcomes in } A$$

- For the Same example:

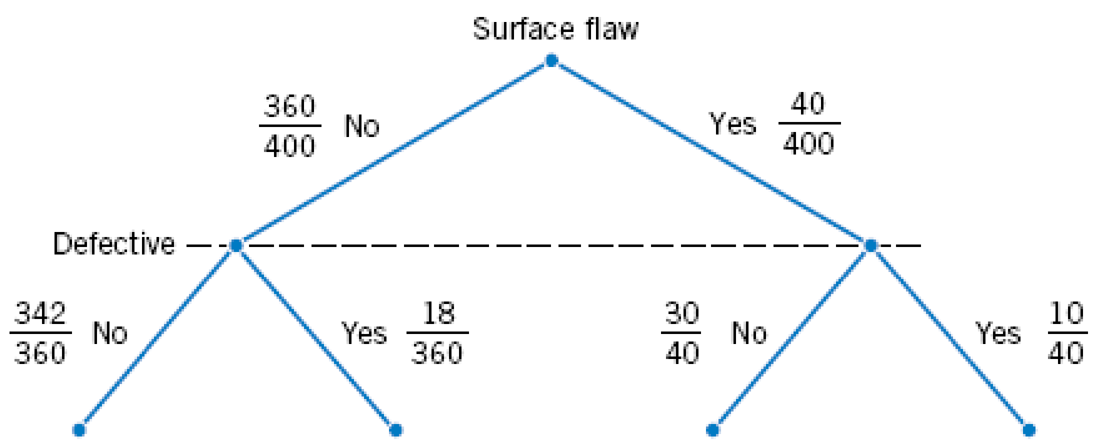
$P(D|F) = P(D \cap F)/P(F) = (10/400)/(40/400) = 10/40$

Note that,

$P(F) = 40/400$ $\quad$ $P(F|D) = 10/28$

$P(D) = 28/400$ $\quad$ $P(D|F) = 10/40$

- Tree Diagram Can be used as follows:



$P(D|F) = 10/40$

$P(D'|F) = 30/40$

$P(D|F') = 18/360$

$P(D'|F') = 342/360$

- **Question 2-79.**

		coating weight	
		high	low
surface	high	12	16
roughness	low	88	34

a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?

Probability = 12/(12+88) = 12/100

b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?

Probability = 12/(12+16) = 12/28

c) If the surface roughness of a sample is low, what is the probability that the coating weight is low?

Probability = 34/(34+88) = 34/122

- **Question 2-83.**

A lot of 100 semiconductor chips contains 20 that are defective. Two are selected randomly, without replacement, from the lot.

(a) What is the probability that the first one selected is defective?

P(1st is defective) = 20/100

(b) What is the probability that the second one selected is defective given that the first one was defective?

P(2nd is defective|1st is defective) = 19/99

(c) What is the probability that both are defective?

P(both are defective) = (20/100)*(19/99) = 0.038

(d) How does the answer to part (b) change if chips selected were replaced prior to the next selection?

P(2nd is defective|1st is defective) = 20/100

2-5 MULTIPLICATION AND TOTAL PROBABILITY RULES:

2-5.1 Multiplication Rule:

$$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$$

- **Example:**

The probability that an automobile battery subject to high engine compartment temperature suffers low charging current is 0.7. The probability that a battery is subject to high engine compartment temperature is 0.05.

C: denote the event that a battery suffers low charging current

T: denote the event that a battery is subject to high engine compartment temperature

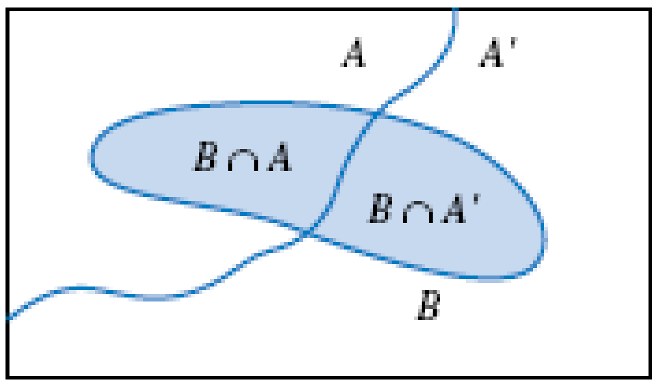
$$P(C \cap T) = P(C|T)P(T) = 0.7 \times 0.05 = 0.035$$

2-5.2 Total Probability Rule:

❖ Sometimes the probability of an event is given under each of several conditions.

❖ For any event **B**, we can write it as follows:

$$B = (A \cap B) \cup (A' \cap B)$$



❖ From the probability of the union of mutually exclusive events and multiplication rule, we have the following **total probability rule**:

For any events A and B,

$$P(B) = P(B \cap A) + P(B \cap A') = P(B|A)P(A) + P(B|A')P(A')$$

➢ Example:

Semiconductor Contamination Example

Probability of Failure	Level of Contamination	Probability of Level
0.1	**High**	**0.2**
0.005	**Not High**	**0.8**

F: The product fails

H: High level of contamination

$$P(F|H) = 0.10 \quad \text{and} \quad P(F|H') = 0.005$$
$$P(H) = 0.20 \quad \text{and} \quad P(H') = 0.80$$

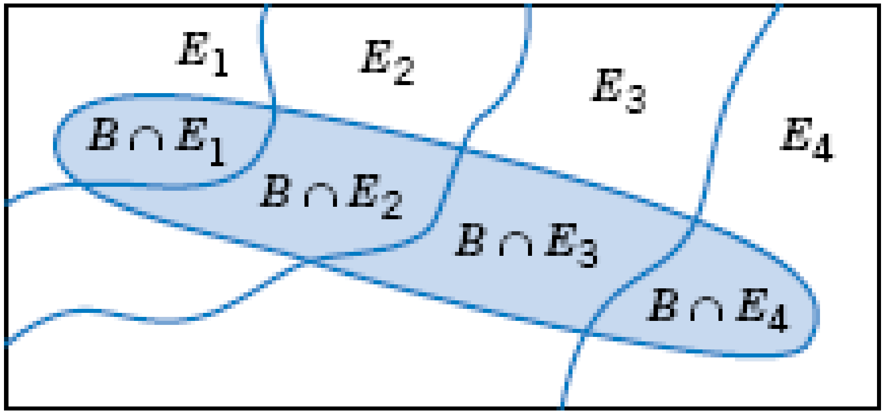
So that, $P(F) = P(F \cap H) + P(F \cap H') = P(F|H)*P(H) + P(F|H')*P(H')$

$= 0.1(0.2) + 0.005(0.8) = 0.024$

❖ For Multiple Events:

Assume $E_1, E_2, \ldots, E_k$ are k mutually exclusive and exhaustive sets. Then

$$\begin{aligned} P(B) &= P(B \cap E_1) + P(B \cap E_2) + \cdots + P(B \cap E_k) \\ &= P(B \mid E_1)P(E_1) + P(B \mid E_2)P(E_2) + \cdots + P(B \mid E_k)P(E_k) \end{aligned}$$



➢ Example:

Semiconductor Contamination Example

Probability of Failure	Level of Contamination
0.10	High
0.01	Medium
0.001	Low

F: The product fails
H: High level of contamination
M: Medium level of contamination
L: Low level of contamination

$P(H) = 0.2$ $\quad P(M) = 0.3$ $\quad P(L) = 0.5$

So that,

$$P(F) = P(F \cap H) + P(F \cap M) + P(F \cap L) = P(F|H)*P(H) + P(F|M)*P(M) + P(F|L)*P(L)$$
$$= 0.1(0.2) + 0.01(0.3) + 0.001(0.5) = 0.0235$$

Question 2-93.

The probability is **1%** that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is **5%**. If **90%** of the connectors are kept dry and **10%** are wet, what proportion of connectors fail during the warranty period?

F: Event of failure
D: Electrical connector is dry
W: Electrical connector is wet

$$P(F) = (F \cap D) + P(F \cap W) = P(F|D)P(D) + P(F|W)P(W)$$
$$= 0.01(0.9) + 0.05(0.1) = 0.009 + 0.005 = 0.014$$

2-6 INDEPENDECE:

❖ In some cases the conditional probability of $P(B|A)$ might equal $P(B)$.

❖ the outcome of the experiment is in event A does not affect the probability that the outcome is in event B.

➢ **Example:**

- 850 manufactured parts contains 50 parts that do not meet customer requirements.

-Two parts are selected from the batch, but the first part is replaced before the second part is selected.

-What is the probability that the second part is defective (denoted as B) given that the first part is defective (denoted as A)?

$$P(B|A) = 50/850 \text{ (with replacement)}$$

Then

$$P(A\cap B) = P(B|A)*P(A) = (50/850)*(50/850) = 0.0035$$

➤ Example:

		Surface Flaws		
		Yes (event F)	No	Total
Defective	Yes (event D)	2	18	20
	No	38	342	380
	Total	40	360	400

$P(D|F) = 2/40 = 0.05$ and $P(D) = 20/400 = 0.05$

The probability of F does not depend on whether it has a surface flaws.

$P(F|D) = 2/20 = 0.10$ and $P(F) = 40/400 = 0.10$

Te probability of a surface flow does not depend on whether the part is defective.

Then we have,

$P(F \cap D) = P(D|F)P(F) = P(D)P(F) = (2/40)*(2/20) = 1/200$

❖ IF P(B|A) = P(B), we can conclude that

$$P(A \cap B) = P(B|A)P(A) = P(B)P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

❖ Independence (two events):

Two events are **independent** if any one of the following equivalent statements is true:

(1) $P(A|B) = P(A)$

(2) $P(B|A) = P(B)$

(3) $P(A \cap B) = P(A)P(B)$

- **Example:**

\- 850 manufactured parts contains 50 parts that do not meet customer requirements.

A: the first part is defective
B: the second part is defective

P(B|A) = 49/849

But

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|A')P(A') \\ &= (49/849)(50/850) + (50/849)(800/850) \\ &= 50/850 \end{aligned}$$

Because P(B|A) ≠ P(B) → ***not independent***

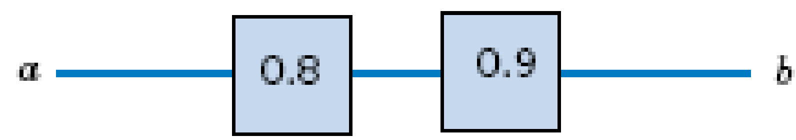
❖ Independence (multiple events):

The events $E_1, E_2, \ldots, E_n$ are independent if and only if for any subset of these events $E_{i_1}, E_{i_2}, \ldots, E_{i_k}$,

$$P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times \cdots \times P(E_{i_k})$$

❖ This definition is typically used to calculate the probability that several events occur assuming that they are independent and the individual event probabilities are known. The knowledge that the events are independent usually comes from a fundamental understanding of the random experiment.

- **Example:**

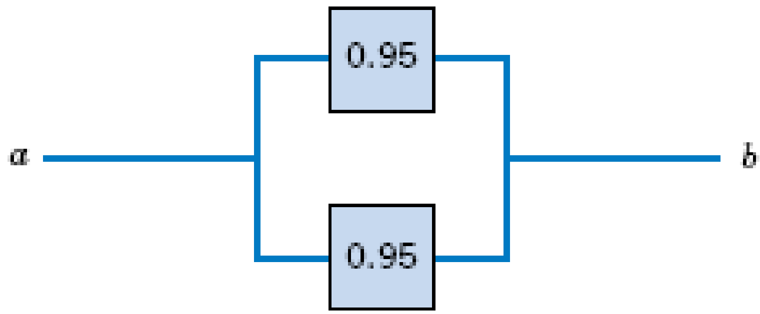


The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?

L: Left device operates
R: Right device operates

$P(L \cap R) = P(L)P(R) = 0.8(0.9) = 0.72$

➢ **Example:**



The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?

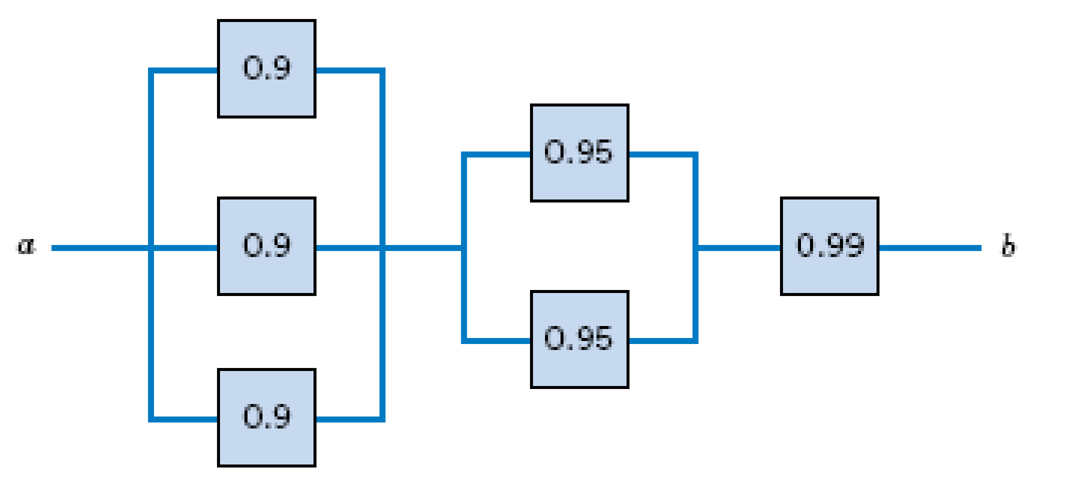
T: Top device operates
B: Bottom device operates

$P(T \cup B) = P(T) + P(B) - P(T \cap B) = 0.95 + 0.95 - 0.95(0.95) = 0.9975$

Another method,

$P(T \cup B) = 1 - P[(T \cup B)'] = 1 - P(T' \cap B') = 1 - [P(T')P(B')] = 1 - [(1-0.95)*(1-0.95)$
$= 0.9975$

➢ Example:



For the first column:

Probability = $1 – (1\text{-}0.9)^3 = 0.999$

For the second column:

Probability = $1 – (1\text{-}0.95)^2 = 0.9975$

For the third column:

Probability = 0.99

Then Total Probability = (0.999)(0.9975)(0.99) = 0.9865

➢ Question 2-105.

100 Samples from 3 Suppliers.

		conforms	
		yes	no
	1	22	8
supplier	2	25	5
	3	30	10

A: Sample is from supplier 1
B: Sample conforms to specifications

a) Are A & B Independent?????

$P(A) = (22+8)/100 = 30/100 = 0.3$
$P(A|B) = 22/(22+25+30) = 22/77 = 0.286$

$P(A) \neq P(A|B)$ NO (Not Independent)

b) **P(B|A)** $= 22/30 = 0.733$

➢ Question 2-109.

Six tissues are extracted.
The plant is infested in 20% of its area.
A Sample is chosen from randomly selected areas on the ivy plant.
Assume that the samples are independent.

A) What is the probability that four successive samples show the signs of infestation???

Probability = (0.2*0.2*0.2*0.2) * 3 = 0.0048

B) What is the probability that three out of four successive samples show the signs of infestation???

Probability = (0.2*0.2*0.2*0.8) * 4 *3 = 0.0768

2-7 BAYES' THEOREM:

❖The conditional probability provide the probability of an event given a condition. (Failure | High or Low Contamination).

❖ After a random experiment generates an outcome, we are naturally interested in the probability that a condition was present (high contamination) given an outcome (failure).

❖ We know that:

$$P(A \cap B) = P(A|B)P(B) = P(B \cap A) = P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad \text{for} \quad P(B) > 0}$$

This is a useful result that enables us to solve for P(A|B) in terms of P(B|A).

➢ Example:

Semiconductor Contamination Example

Probability of Failure	Level of Contamination	Probability of Level
0.1	High	0.2
0.005	Not High	0.8

$$P(F) = P(F \cap H) + P(F \cap H') = P(F|H)*P(H) + P(F|H')*P(H')$$
$$= 0.1(0.2) + 0.005(0.8) = 0.024$$

$$P(H|F) = [P(F|H)*P(H)] / P(F) = (0.1*0.2)/0.024 = 0.83$$

❖ Bayes' Theorem

Bayes' Theorem

If $E_1, E_2, \ldots, E_k$ are k mutually exclusive and exhaustive events and B is any event,

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + \cdots + P(B|E_k)P(E_k)}$$

$$\text{for } P(B) > 0$$

➢ Example:

-The probability that the test correctly identifies someone with the illness as positive is 0.99

-The probability that the test correctly identifies someone without the illness as negative is 0.95

-The incidence of the illness in the general population is 0.0001

-You take the test, and the result is positive. What is the probability that you have the illness?

D: you have the illness
S: test signals positive

$$P(D|S) = P(S|D)P(D) / [P(S|D)P(D) + P(S|D')P(D')]$$
$$= 0.99(0.0001) / [0.99(0.0001) + (1-0.95)(1-0.0001)] = 1/506 = 0.002$$

➢ Example:

- Printer failures are associated with three types of problems: Hardware, Software, and Other, with probabilities 0.1, 0.6, and 0.3, respectively.

- The probability of printer failure given a hardware problem is 0.9, given s software problem is 0.2, and given any other type of problem is 0.5.

- If a customer enters the manufacture's website to diagnose a printer failure, what is the most likely cause of the problem?

$P(H) = 0.1, P(S) = 0.6, P(O) = 0.3$
$P(F|H) = 0.9, P(F|S) = 0.2, P(F|O) = 0.5$

$P(F) = (F|H)P(H) + P(F|S)P(S) + P(F|O)P(O) = 0.9*0.1 + 0.2*0.6 + 0.5*0.3 = 0.36$

$P(H|F) = P(F|H)P(H) / P(F) = 0.9(0.1)/0.36 = 0.25$
$P(S|F) = P(F|S)P(S) / P(F) = 0.2(0.6)/0.36 = 0.333$
$P(O|F) = P(F|O)P(O) / P(F) = 0.5(0.3)/0.36 = 0.417$ ***(MOST LIKELY)***

2-8 RANDOM VARIABLES:

❖ Because the particular outcome of the experiment is not known in advance, the resulting value of our variable is not known in advance. For this reason, the variable that associates a number with the outcome of a random experiment is referred to as a **random variable**.

Definition

> A **random variable** is a function that assigns a real number to each outcome in the sample space of a random experiment.
>
> A random variable is denoted by an uppercase letter such as X. After an experiment is conducted, the measured value of the random variable is denoted by a lowercase letter such as $x = 70$ milliamperes.

❖ Discrete and Continuous Random Variables:

A **discrete** random variable is a random variable with a finite (or countably infinite) range.
A **continuous** random variable is a random variable with an interval (either finite or infinite) of real numbers for its range.

Examples of **continuous** random variables:
electrical current, length, pressure, temperature, time, voltage, weight

Examples of **discrete** random variables:
number of scratches on a surface, proportion of defective parts among 1000 tested, number of transmitted bits received in error.

➢ Question 2-121.

Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

a) What is the probability that a product attains a good review??

G: a product that received a good review
H, M, and P denote products that were high, moderate, and poor performers, respectively.

$$P(G) = P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) = 0.95(0.4)+0.6(0.35)+0.1(0.25)$$
$$= 0.615$$

b) If a new design attains a good review, what is the probability that it will be a highly successful product?

$$P(H|G) = [P(G|H)P(H)]/P(G) = 0.95(0.4)/0.615 = 0.618$$

c) If a product does not attain a good review, what is the probability that it will be a highly successful product?

$$P(H|G') = [P(G'|H)P(H)]/P(G') = (1-0.95)(0.4)/(1-0.615) = 0.02/0.385 = 0.052$$